Journal of Applied Mechanics and Technical Physics, Vol. 42, No. 4, pp. 701–703, 2001

NONLINEAR MODES OF MOTION OF THIN CIRCULAR CYLINDRICAL SHELLS

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UDC 539.3:534.1

Free flexural vibrations of a simply supported shell are studied within the framework of the nonlinear theory of flexible shallow shells. It is assumed that large-amplitude flexural vibrations are coupled with radial vibrations of the shell. Modal equations are derived by the Bubnov–Galerkin method. Periodic solutions are obtained by the Krylov–Bogolyubov method. The skeleton curve of the soft type obtained using a nonlinear finite-dimensional shell model agrees with available experimental data.

Introduction. Despite the fact that the nonlinear dynamics of shells has been the subject of myriad studies [1], a number of problems remain to be solved. In particular, the problem of large-amplitude flexural vibrations of a simply supported shell of finite length has not been solved. Within the framework of traditional nonlinear finite-dimensional shell models [1–5], which give a skeleton curve of the soft type, the boundary condition for the bending moment is not satisfied. Therefore, these models are valid only for long shells. Attempts to solve this problem with allowance for all boundary conditions were not successful [2, 3]. These models give a skeleton curve of the rigid type, which does not agree with available experimental data.

In the present paper, a nonlinear finite-dimensional shell model that gives a skeleton curve of the soft type is proposed, and the problem of nonlinear flexural vibrations of a simply supported shell of finite length is solved.

Equations of Motion. The mathematical model is based on the nonlinear relations for flexible shallow shells [5]:

$$\frac{1}{E}\,\nabla^4\Phi = -\frac{1}{2}\,L(w,w) - \frac{1}{R}\,\frac{\partial^2 w}{\partial x^2}, \qquad \frac{D}{h}\,\nabla^4 w = L(w,\Phi) + \frac{1}{R}\,\frac{\partial^2 \Phi}{\partial x^2} - \rho\,\frac{\partial^2 w}{\partial t^2}. \tag{1}$$

Here ∇^4 and L are differential operators, $w(x,y,t)$ is the elastic deflection, $\Phi(x,y,t)$ is a stress function in the mid-surface, R is the radius of the shell, $D = Eh^3/(12(1-\mu^2))$ is the flexural rigidity, E is Young's modulus, h is the thickness, μ is Poisson's ratio, ρ is the density, and t is time.

We consider a shell of length l which is simply supported in such a manner that the points of its boundary contour can move in the longitudinal and circumferential directions:

$$w = \frac{\partial^2 w}{\partial x^2} = N_x = T = 0 \quad \text{for} \quad x = 0, \quad x = l. \tag{2}$$

Here N_x and T are the specific longitudinal and tangential forces, respectively.

Nonlinear Vibration Modes. The deflection of the shell vibrating with fundamental frequency can be approximated by the following expression [1–5]:

$$w(x,y,t) = h\{[f_1(t)\sin(\beta y) + f_2(t)\cos(\beta y)]\sin(\alpha x) + \Psi(x,t)\}, \quad \alpha = \pi/l, \quad \beta = n/R. \tag{3}$$

Here n is the number of waves in the circumferential direction, and the conjugate flexural modes $\sin(\beta y)\sin(\alpha x)$ and $\cos(\beta y)\sin(\alpha x)$ are small-amplitude vibration modes of the shell.

It is well known that the nonlinear behavior of a shell depends strongly on the axisymmetric component of the deflection $\Psi(x,t)$, which is taken in the form $\Psi(x,t) = f_3(t)\sin^2(\alpha x)$ [1–5] to construct the finite-dimensional model (3). But using this approach, based on geometrical model assumptions of shell deformation ("predominant inward buckling" and inextensible contour), one cannot satisfy boundary conditions (2) and, hence, obtain a reasonably accurate solution for a shell of finite length.

Komsomol'sk-on-Amur State Technical University, Komsomol'sk-on-Amur 681013. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 42, No. 4, pp. 161–164, July–August, 2001. Original article submitted February 12, 1999; revision submitted March 13, 2001.

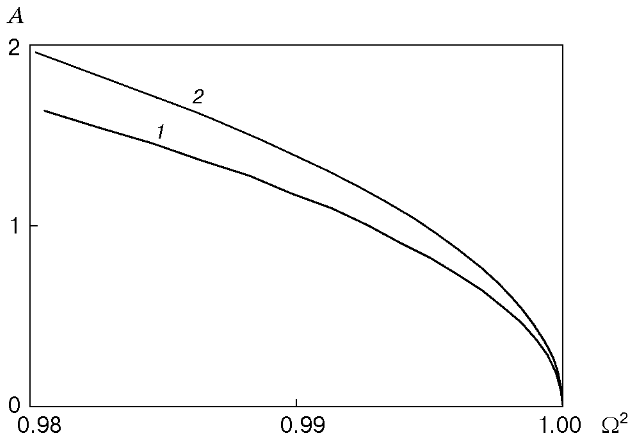

Fig. 1

At the same time, the use of the function $\Psi(x,t) = f_3(t)\sin(\alpha x)$ which satisfies all the simply-supported boundary conditions is not recommended [1–3] since this leads to rigid single-mode motion of the shell.

In this paper, we propose a different approach to constructing the nonlinear finite-dimensional model (3). We assume that large-amplitude flexural vibrations are coupled with radial vibrations of the shell (in a linear formulation, these vibrations are known to be independent). The axisymmetric part of the deflection $\Psi(x,t)$ can be obtained by "summation" of the modes of small radial vibrations. Retaining two terms in this expansion, we write the deflection in the form

$$w(x,y,t) = h\{[f_1(t)\sin(\beta y) + f_2(t)\cos(\beta y)]\sin(\alpha x) + f_3(t)\sin(\alpha x) + f_4(t)\sin(3\alpha x)\}.$$

Modal Equations. We solve Eqs. (1) using the procedure proposed by P. F. Papkovich. We first determine the stress function Φ. As in [2–5], the tangential boundary conditions $N_x = T = 0$ are satisfied "on average." Using the Bubnov–Galerkin method, we obtain a system of four nonlinear differential equations for the generalized coordinates. Next, the system is simplified by analogy with [2, 5]. Finally, the system reduces to two differential equations for the functions f_1 and f_2:

$$\ddot f_1 + f_1 + c_1 f_1(\ddot f_1 f_1 + \ddot f_2 f_2 + \dot f_1^2 + \dot f_2^2) + c_2 f_1(f_1^2 + f_2^2) = 0,$$

$$\ddot f_2 + f_2 + c_1 f_2(\ddot f_1 f_1 + \ddot f_2 f_2 + \dot f_1^2 + \dot f_2^2) + c_2 f_2(f_1^2 + f_2^2) = 0.$$

Here dots denote differentiation with respect to the nondimensional time $\tau = \lambda t$ (λ is the eigenfrequency),

$$c_1 = \frac{32\varepsilon}{9\pi^2 p_1^2}\Big[\Big(1 + \frac{2\theta^4}{(1+\theta^2)^2}\Big)\Big(1 + \frac{\theta^4}{(1+\theta^2)^2} + \frac{\theta^4}{(1+4\theta^2)^2}\Big)$$

$$+ \frac{p_1^2}{25p_2^2}\Big(1 + \frac{18\theta^4}{(1+\theta^2)^2}\Big)\Big(1 + \frac{9\theta^4}{(1+\theta^2)^2} + \frac{45\theta^4}{(1+4\theta^2)^2} - \frac{36\theta^4}{(1+16\theta^2)^2}\Big)\Big],$$

$$c_2 = \frac{\varepsilon}{16\omega^2}\Big(3 + \theta^4 - \frac{8}{\varepsilon}c_1\Big), \qquad p_1^2 = 1 + \frac{\varepsilon\theta^4}{12(1-\mu^2)}, \qquad p_2^2 = 1 + \frac{81\varepsilon\theta^4}{12(1-\mu^2)},$$

$$\omega^2 = \frac{\rho R^2}{E}\lambda^2 = \frac{\varepsilon(1+\theta^2)^2}{12(1-\mu^2)} + \frac{\theta^4}{(1+\theta^2)^2}, \qquad \varepsilon = \Big(\frac{n^2 h}{R}\Big)^2, \qquad \theta = \frac{\pi R}{nl}.$$

Results and Conclusions. The skeleton curves corresponding to a standing wave [$f_1 = A\cos(\Omega\tau)$ and $f_2 \equiv 0$] and a traveling wave [$f_1 = A\cos(\Omega_{\rm f}\tau)$ and $f_2 = A\sin(\Omega_{\rm f}\tau)$] are defined by the following equations, obtained by the Krylov–Bogolyubov method:

$$\Omega^2 = (\tilde\omega/\omega)^2 = 1 - (2c_1 - 3c_2)A^2/4, \qquad \Omega_{\rm f}^2 = (\tilde\omega_{\rm f}/\omega)^2 = 1 + c_2 A^2. \tag{4}$$

Here $\tilde\omega$ and $\tilde\omega_{\rm f}$ are the nondimensional frequencies of free nonlinear vibrations.

To verify the above relations, we consider the limiting case of an infinitely long shell. As $\theta \to 0$, $c_1 = 832\varepsilon/(225\pi^2) \approx 3\varepsilon/8$, $c_2 = 9(1 - 6656/(675\pi^2))/4 \approx 0$, $\Omega^2 = 1 - 3\varepsilon A^2/16$, and $\Omega_{\rm f}^2 = 1$. These relations are equivalent to those given in [6].

Figure 1 shows the skeleton curves of the soft type for the single-mode motion of a relatively long shell with parameters $l/R = 2.5$ and $R/h = 320$ for $\mu = 0.3$ and $n = 6$ ($\varepsilon = 0.0127$ and $\theta = 0.21$). Curve 1 is calculated by the first formula in (4) and curve 2 shows the results of [2], where the axisymmetric deflection component is taken in the form $\Psi(x,t) = f_3(t)\sin^2(\alpha x)$. Curve 2 lies above curve 1 since the traditional approach overestimates the generalized rigidity of a simply supported shell. Calculations show that the error of the solution obtained by the traditional approach increases considerably as the relative length of the shell decreases.

In conclusion, we consider the approximation of the deflection by the function $\Psi(x,t) = f_3(t)\sin(\alpha x)$ [$f_4(t) \equiv 0$]. In this case, as $\theta \to 0$, $c_1 = 32\varepsilon/(9\pi^2)$ and the coefficient $c_2 = 9(1 - 256/(27\pi^2))/4$ becomes nonzero and takes on a positive value. The difference $1 - 256/(27\pi^2) \approx 0.04$ is due to the error of the first approximation for the radial vibration mode. For this reason, the skeleton curve obtained in [2, 3] for relatively long shells is of the rigid type.

An analysis shows that the results obtained using the above-proposed approach to constructing a nonlinear dynamic finite-dimensional shell model agree with the theoretical and experimental data [1–5] obtained for relatively long shells. In contrast to the traditional approach, this approach satisfies all boundary conditions of the problem, including the tangential boundary conditions; therefore, it can be used to calculate the nonlinear dynamic characteristics of shells of arbitrary length.

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